Measurement of $K^+ \to \mu^+ \nu_\mu \gamma$ decay form factors in OKA experiment

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on behalf of OKA collaboration

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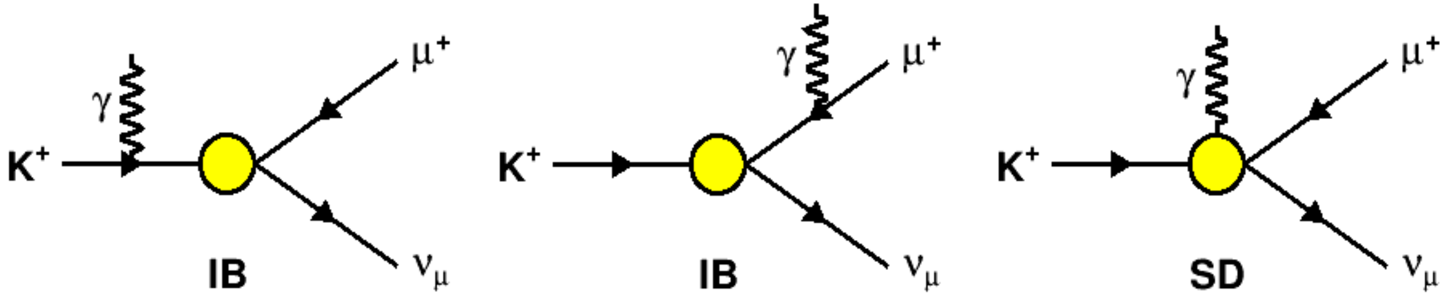
$K \to \mu\nu_\mu\gamma$ decay



IB - inner bremsstrahlung, where photon is emitted from the charged particle in the initial or final state

SD - structure-dependent radiative decay, which involves the emission of a photon from the intermediate states in the transition

INT - possible interference of **IB** and **SD**

Differential cross section in K-meson rest frame

$$\frac{d\Gamma_{K_{\mu\nu\gamma}}}{dxdy} = A_{IB} f_{IB}(x,y) + A_{SD}[(F_V + F_A)^2 f_{SD+}(x,y) + (F_V - F_A)^2 f_{SD-}(x,y)] - A_{INT}[(F_V + F_A) f_{INT+}(x,y) + (F_V - F_A) f_{INT-}(x,y)]$$

where $x = 2E_\gamma/m_K$, $y = 2E_\mu/m_K$, **c.m.s.**

The main goal of analysis is to measure $F_V - F_A$ that connects with INT- and SD-

In lower order of $\chi PT\ O(p^4)$ $F_V = 0.0945$, $F_A = 0.0425$ and $F_V - F_A = 0.052$

First measurement of this difference was made by ISTRA+ (Phys.Lett. B695 (2011) 59-66)

$$F_V - F_A = 0.21 \pm 0.04(stat.) \pm 0.04(syst.)$$

$K \to \mu\nu_\mu\gamma$ decay

$$f_{IB}(x,y) = \left[\frac{1-y+r}{x^2(x+y-1-r)}\right] \times \left[x^2 + 2(1-x)(1-r) - \frac{2xr(1-r)}{x+y-1-r}\right],$$

$$f_{SD^+} = [x+y-1-r][(x+y-1)(1-x)-r],$$

$$f_{SD^-} = [1-y+r][(1-x)(1-y)+r],$$

$$f_{INT^+} = \left[\frac{1-y+r}{x(x+y-1-r)}\right][(1-x)(1-x-y)+r],$$

$$f_{INT^-} = \left[\frac{1-y+r}{x(x+y-1-r)}\right][x^2-(1-x)(1-x-y)-r],$$

$$r = \left[\frac{M_\mu}{M_K}\right]^2,$$

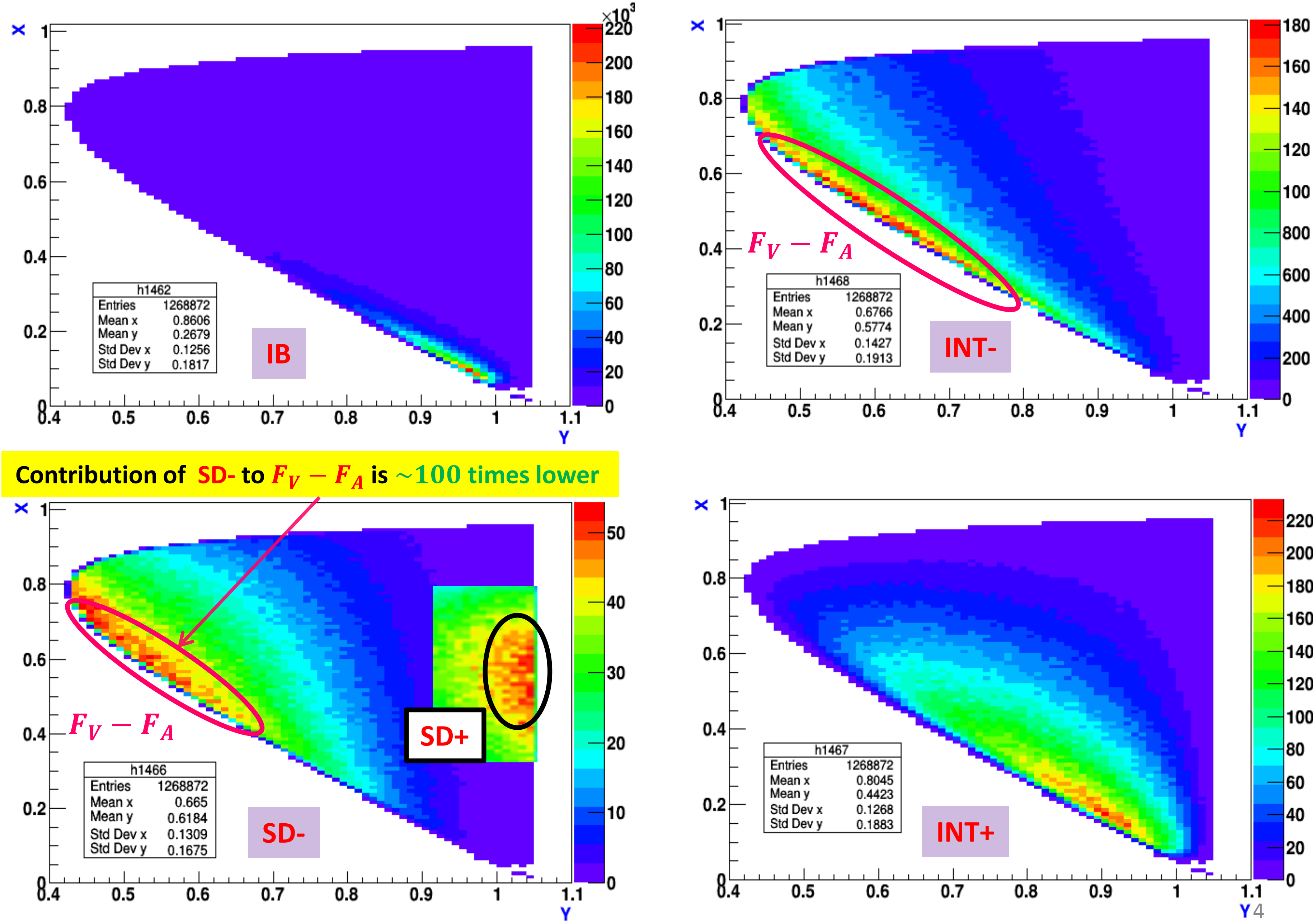
$$A_{IB} = \Gamma_{K_{\mu 2}} \frac{\alpha}{2\pi} \frac{1}{(1-r)^2},$$

$$A_{SD} = \Gamma_{K_{\mu 2}} \frac{\alpha}{8\pi} \frac{1}{r(1-r)^2} \left[\frac{M_K}{F_K}\right]^2,$$

$$A_{INT} = \Gamma_{K_{\mu 2}} \frac{\alpha}{2\pi} \frac{1}{(1-r)^2} \frac{M_K}{F_K}.$$

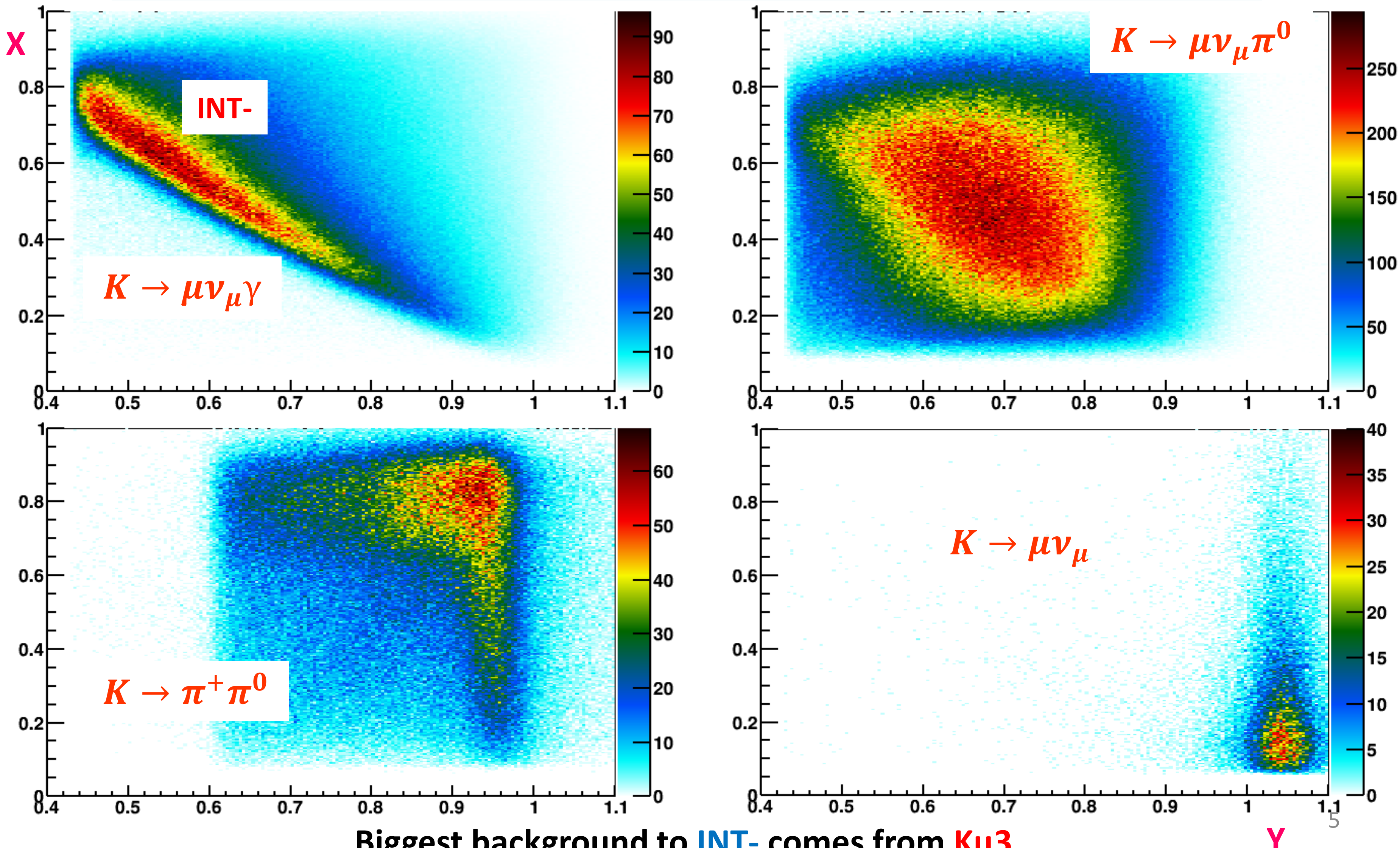
α - fine structure constant,
F_K - K^+decay constant,
$\Gamma_{K_{\mu 2}}$ - width of $K_{\mu 2}$ decay

$K \to \mu\nu_\mu\gamma$ decay matrix



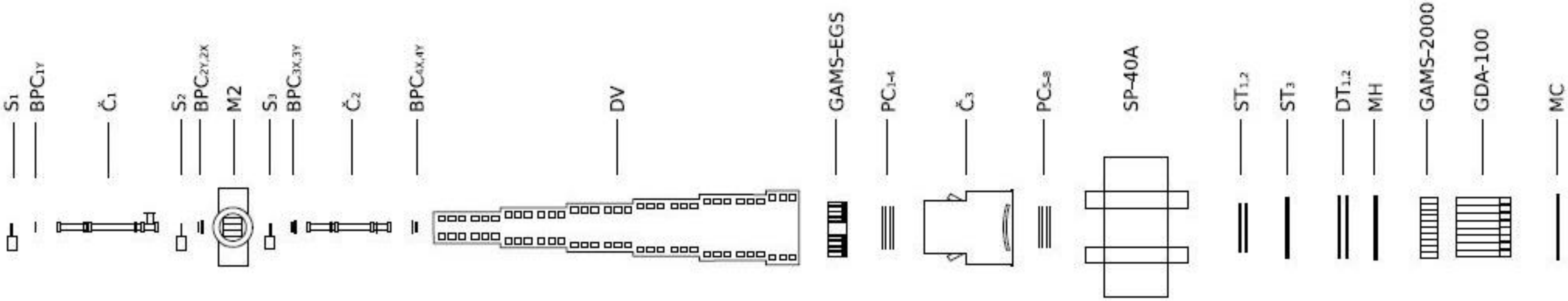
Main backgrounds

$K^+ \to \mu^+\nu_\mu\pi^0$ (**Kμ3**) with $\mathbf{1\gamma}$ lost from $\pi^0\to\gamma\gamma$ (Br = 3.353%)
$K^+ \to \pi^+\pi^0$ (**K2π**) with $\mathbf{1\gamma}$ lost from $\pi^0\to\gamma\gamma$ and $\boldsymbol{\pi}$ misidentification (Br = 20.66%)
$K^+ \to \mu^+\nu_\mu$ with $\mathbf{1\gamma}$ background (Br = 63.55 %)
$K^+ \to \pi^+\pi^- \pi^+$ (**K3π**) with $\mathbf{1\gamma}$ background and $\boldsymbol{\pi}$ misidentification (Br = 5.58%)



Biggest background to INT- comes from Kμ3

OKA setup and event selection



OKA setup includes

Beam spectrometer, **Decay volume** (**DV**) with **Veto** system **, Main magnetic spectrometer,**
2 Gamma detectors (**GAMS-2000**, **EGS**), **Muon identification** (hadron calorimeter **GDA-100** and **MC**),
Matrix Hodoscope (**MH**).

4 Triggers →

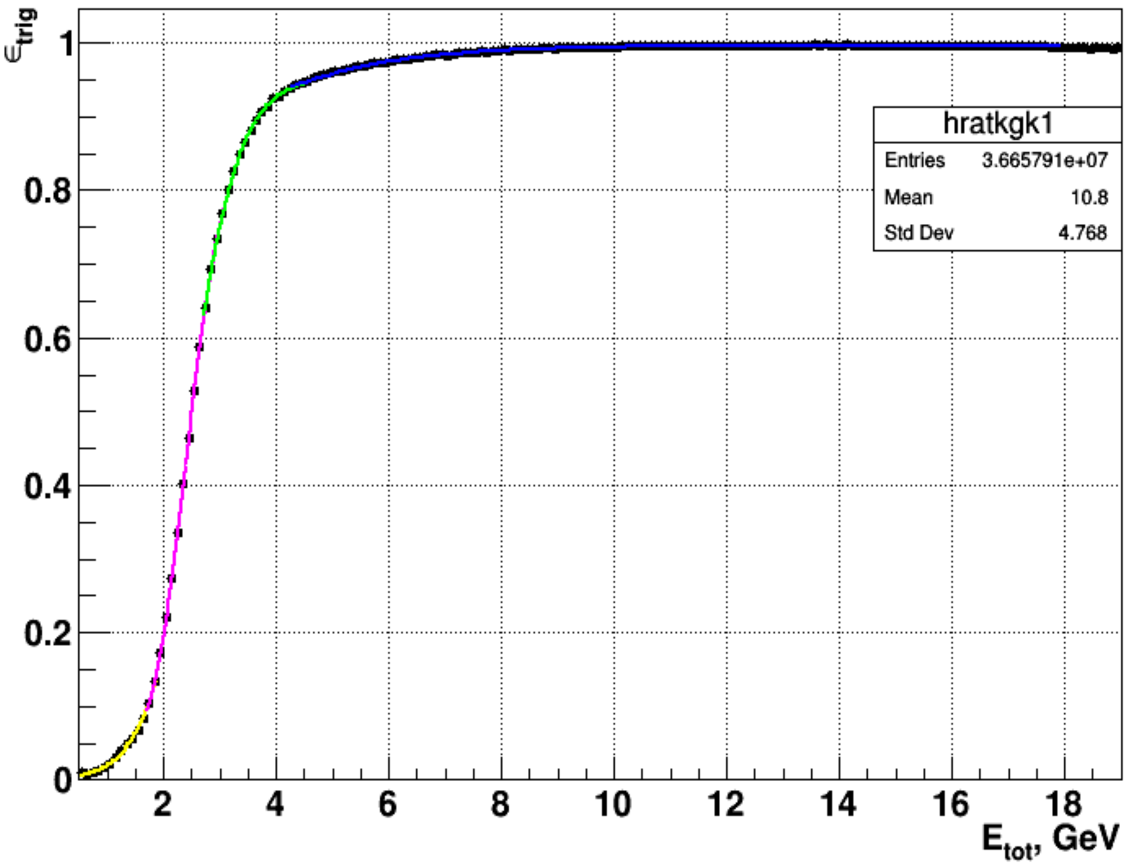
1) **Muon** - $beam * \overline{C_1} * C_2 * \overline{BK} * MC$
2) **Kaon** - $beam * \overline{C_1} * C_2 * \overline{BK}$
3) **GAMS** - $beam * \overline{C_1} * C_2 * \overline{BK} * E_{GAMS}$
4) **$2-4$ tracks** - $beam * C_1 * C_2 * \overline{BK} * MH$

Event selection →

- **GAMS trigger**
- 1 primary beam track
- 1 secondary track identified as **Muon** in **GAMS**, **HCAL** and **MC**
- 1 shower in **GAMS** $> \mathbf{1}\boldsymbol{GeV}$ (not associated with charged track)
- *VETO* calorimeter energy deposition $< \mathbf{10}\boldsymbol{MeV}$
- **GAMS-EGS** energy deposition $< \mathbf{100}\boldsymbol{MeV}$
- **Decay vertex** inside decay volume **DV**

Beam and trigger efficiency

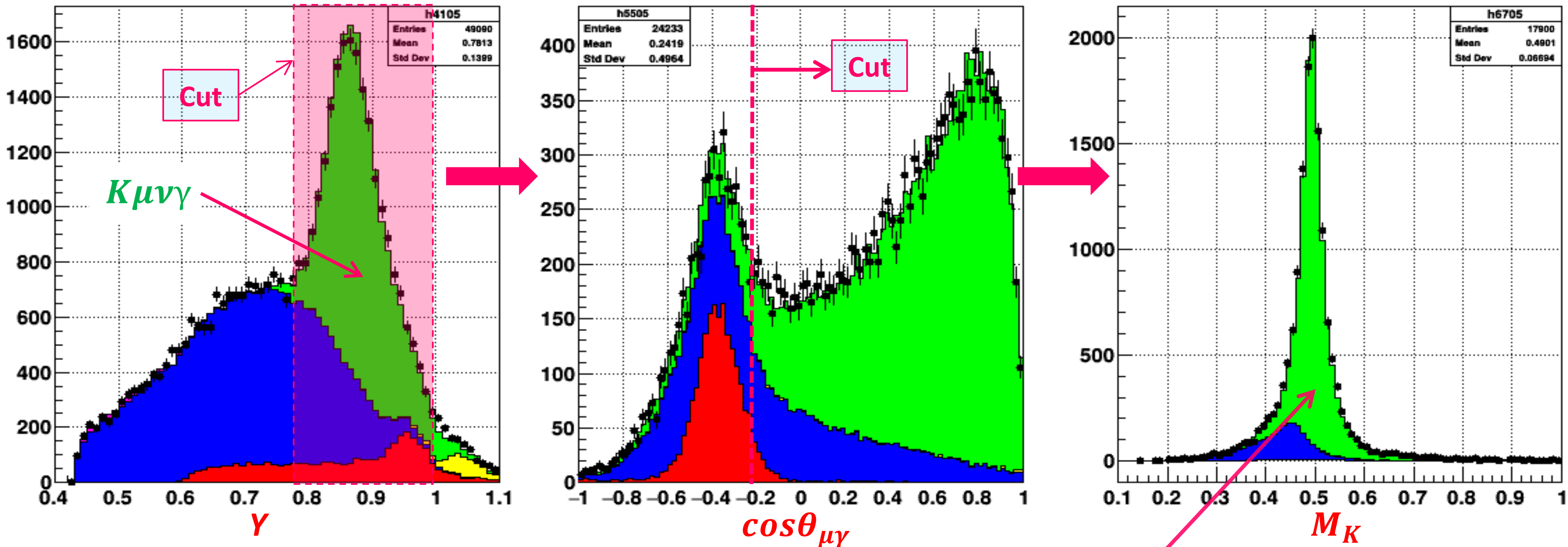
OKA beam is a RF-separated secondary beam of **U-70** Proton Synchrotron of **IHEP, Protvino**.
Beam has up to **12.5%** of kaons with an intensity of $\mathbf{5 \times 10^5}$ kaons per 3 sec U-70 spill.
Beam energy was $\mathbf{17.7\ GeV/c}$ during **analyzed Run 14** (**November 2012**).
The present study uses about **1/2 part** of the statistics taken in 2012,
where **504M** events were stored on tape.



$\boldsymbol{\epsilon_{trig} = (GAMS \cap Kaon)/Kaon}$
"minimum bias " trigger was used for MC simulation.

Color curves - fit by polynomial of the degree three in four intervals.

Method of $K \rightarrow \mu\nu_\mu\gamma$ decay selection



Lab system

$$M_K{}^2 = (P_\mu + P_\nu + P_\gamma)^2$$

P_μ, P_ν, P_γ - 4-momentum of decay particles

$\overrightarrow{p_\nu} = \overrightarrow{p_K} - \overrightarrow{p_\mu} - \overrightarrow{p_\gamma},\ E_\nu = |\overrightarrow{p_\nu}|$

1. All **x-y** kinematical region was divided into **x-stripes** with width $\boldsymbol{\Delta x = 0.05}$.

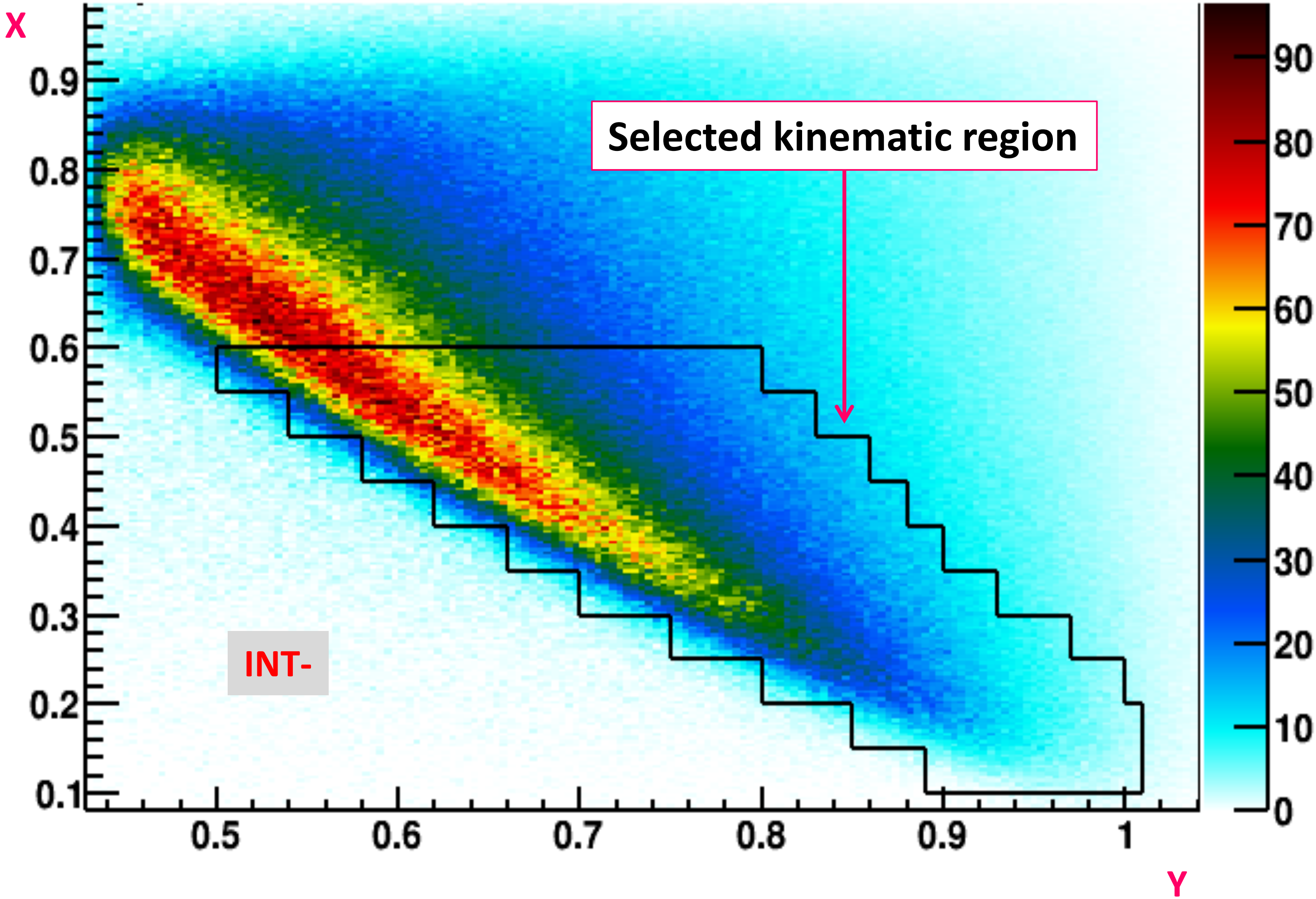
Next steps were applied for each X-stripe:

2. Apply a cut $Y_{min} < Y < Y_{max}$ in signal region and fill $cos\theta_{\mu\gamma}$ plot. $\theta_{\mu\gamma}$ - angle between μ and γ in **c.m.s.**
3. Put a cut on $cos\theta_{\mu\gamma}$ to reject background.
4. Fill M_K plot.
5. For the first iteration **IB** term was used only (Green color).
6. Simultaneous fit of all 3 histograms with **MINUIT.**

Method was proposed by ISTRA+

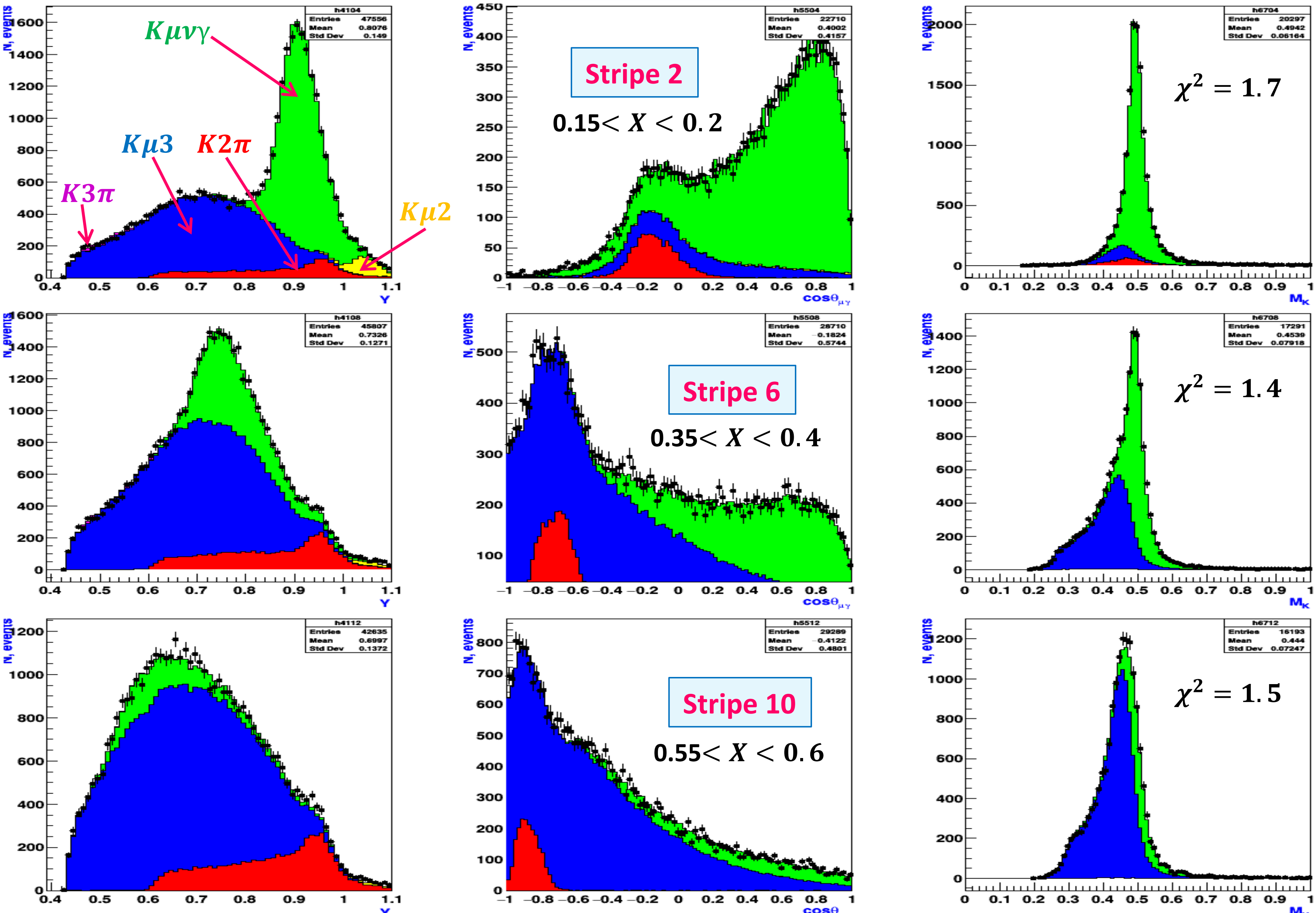
For correct estimation of statistical error σ_{exp} the errors of M_K histogram fit were used .

$K \rightarrow \mu\nu_\mu\gamma$ *decay selection*



The cuts on **Y** for signal in 10 **X-stripes**.

X-*Stripes* 2,6,10



Simultaneous fit has a good agreement with $1.3 < \chi^2/NDF < 1.7$

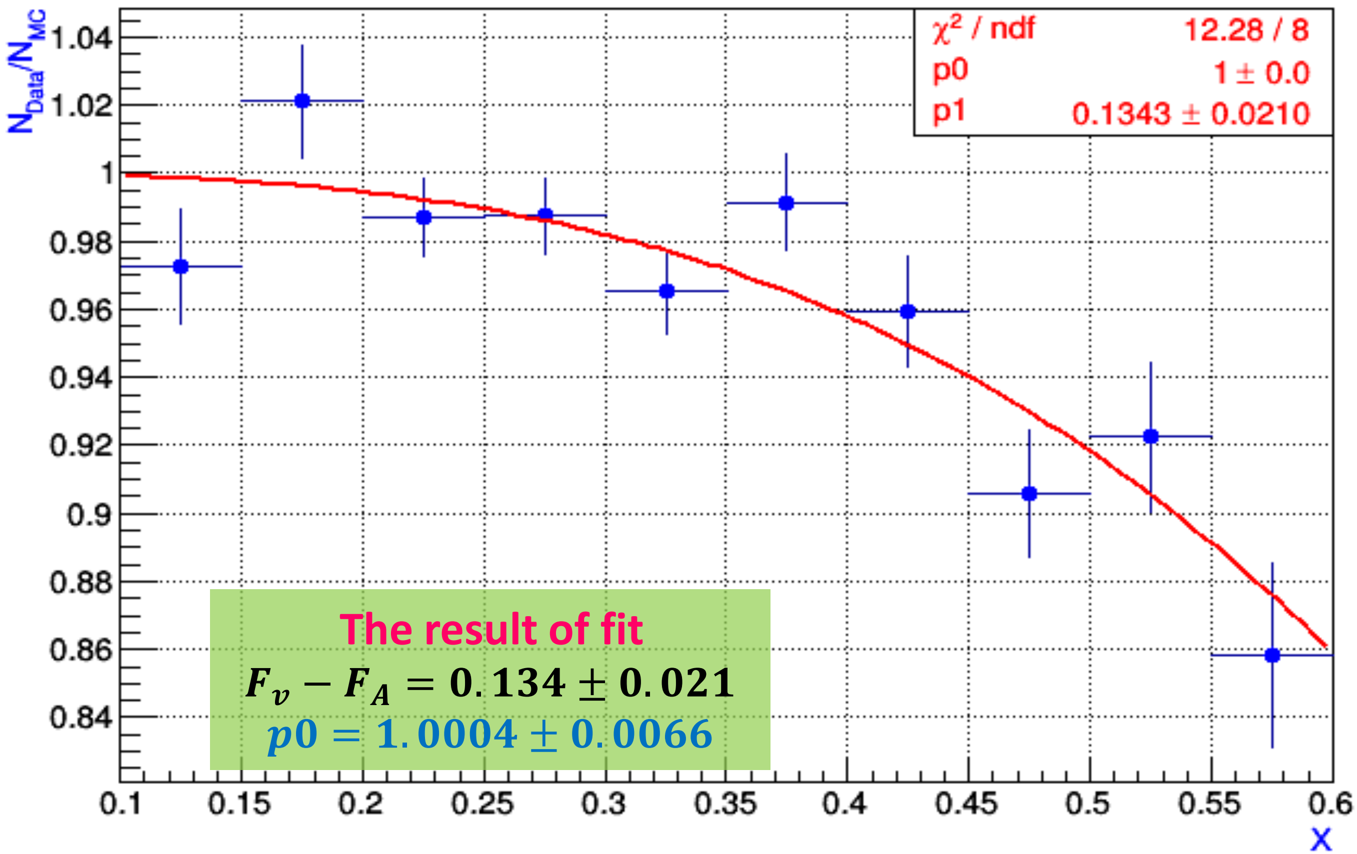
$F_V - F_A$ *calculation*

For each **X-stripe** we have experimental event number N_{Data} from data fitting and **IB** event number N_{IB} from **MC**. Then we plot N_{Data}/N_{IB} as a function of **X**.
For **IB** only we would have N_{Data}/N_{IB} **= 1**. For small **X IB** is dominated and **INT-** is negligible. For large **X** we see that N_{Data} also contains negative interference term.

We fit N_{Data}/N_{IB} distribution with which is a sum of IB and **INT-**

$$p_{signal} = p0[1 + p1(f_{INT-}(x)/f_{IB}(x))]$$

$$p1 = F_v - F_A$$



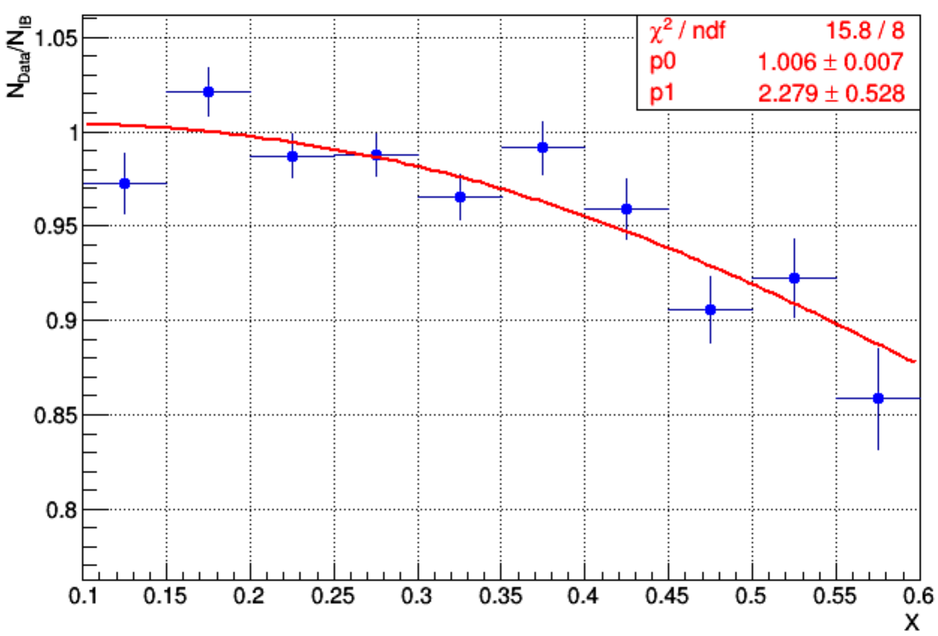
The total number of selected $K \to \mu\nu\gamma$ decay events is $\sim 95K$.

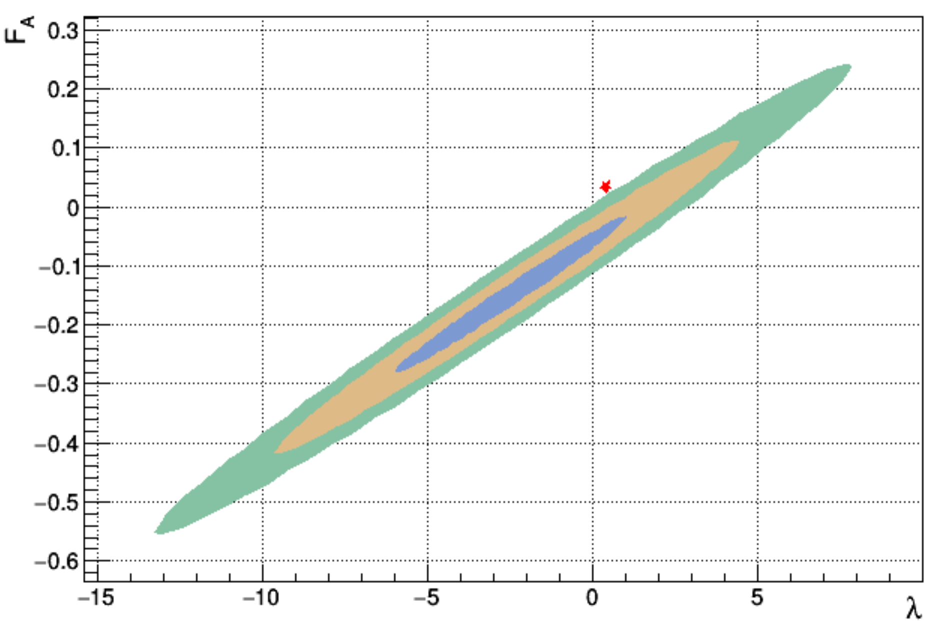
$\chi PT\ O(p^6)$ model

In the next order $\chi PT\ O(p^6)$ F_V depends on the transfer momentum q^2 according to a linear law with parametrization: $F_V = F_V(0)[1+\lambda(1-x)]$. $F_A = const$.

The theoretical prediction was tested in three ways:

1) Final fit with F_V and F_A from $\chi PT\ O(p^6)$: $F_V(0) = 0.082$, $F_A = 0.034$, $\lambda = 0.4$.
   This fit has bad compliance with $\chi^2/NDF = 28.0/9$.
2) $F_V(0)$ and F_A are taken from $\chi PT\ O(p^6)$. It gives $\lambda = 2.28 \pm 0.53$ with $\chi^2/NDF = 15.8/8$.
3) $F_V(0)$ was fixed from $\chi PT\ O(p^6)$. F_A and λ are the fit parameters.
   Fig. shows the $F_A - \lambda$ correlation. Theoretical prediction (**red star**) is slightly out of 3σ-ellipse.



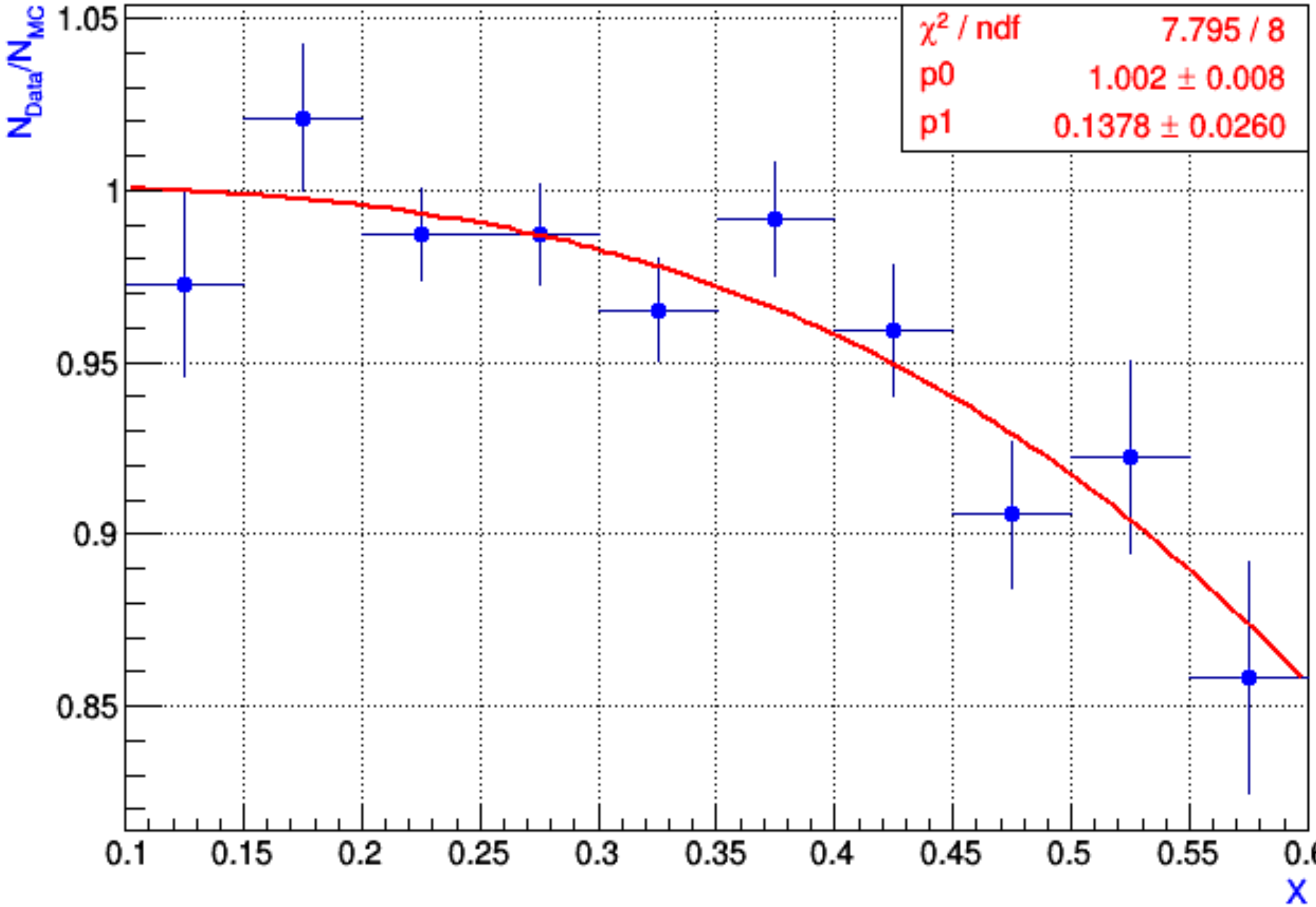


Systematics

Since analysis is complicated and can depends on **width of X-stripes**, **Y** and **angle cuts** and **fit procedure** we try to estimate all possible systematics. Next possibilities are considered:

1) Non ideal *forms of signal and background*

For estimation of systematic error from possible non ideal description of signal and background in **MC**, the error of each bin was scaled by $\sqrt{\chi^2/NDF}$ factor. χ^2 is obtained from simultaneous fit in each **X**-stripe.



Main fit
χ^2/NDF = **1.54**

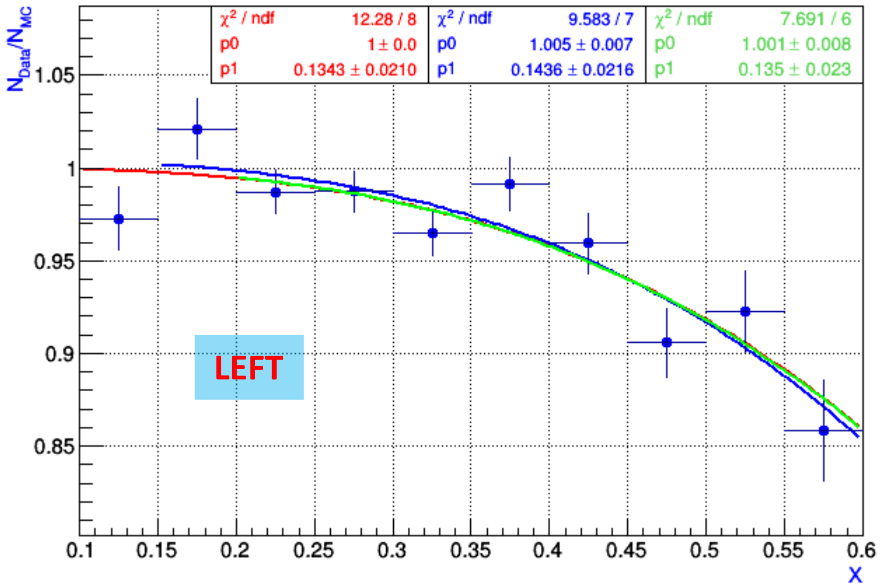
Present fit
χ^2/NDF = **0.97**

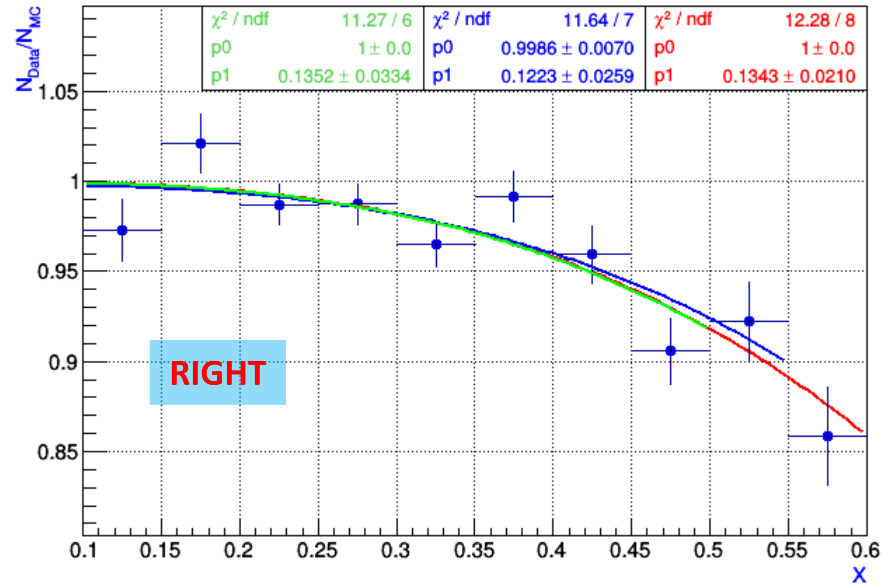
New value of $F_V - F_A$ is consistent with the main one but the fit error is larger. We suppose σ_{form} depends as $\sigma^2{}_{fit} = \sigma^2{}_{shape} + \sigma^2{}_{stat}$ and therefore

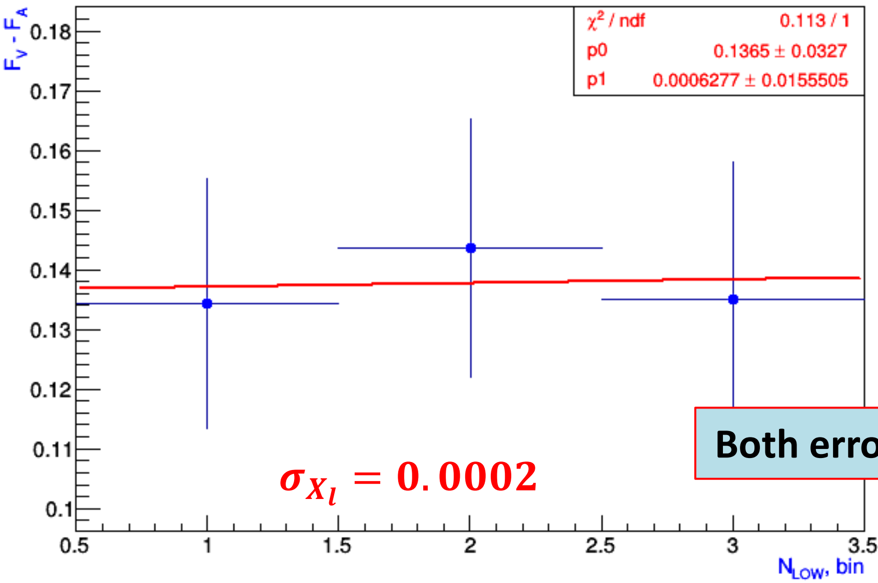
$\sigma_{stat} = 0.021$ → $\sigma_{shape} = 0.0153$

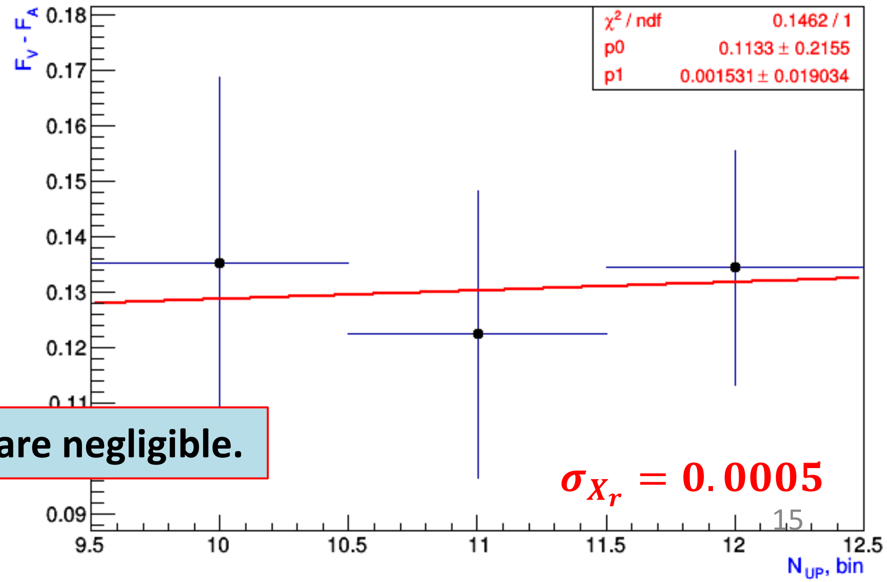
2) Left and right X limits

Dependency N_{Data}/N_{IB} on **X** was fitted by removing points at the left (right) edge.
The result points were fitted by straight line for conservative estimate of systematics.
The line slope multiplied by the resolution in **X** (from **MC**) gives systematic error.



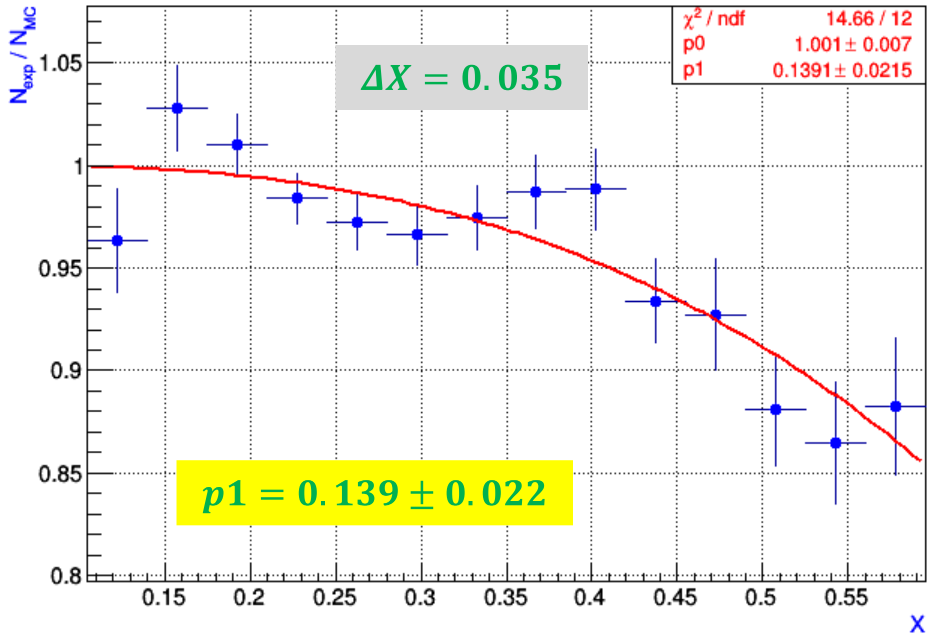


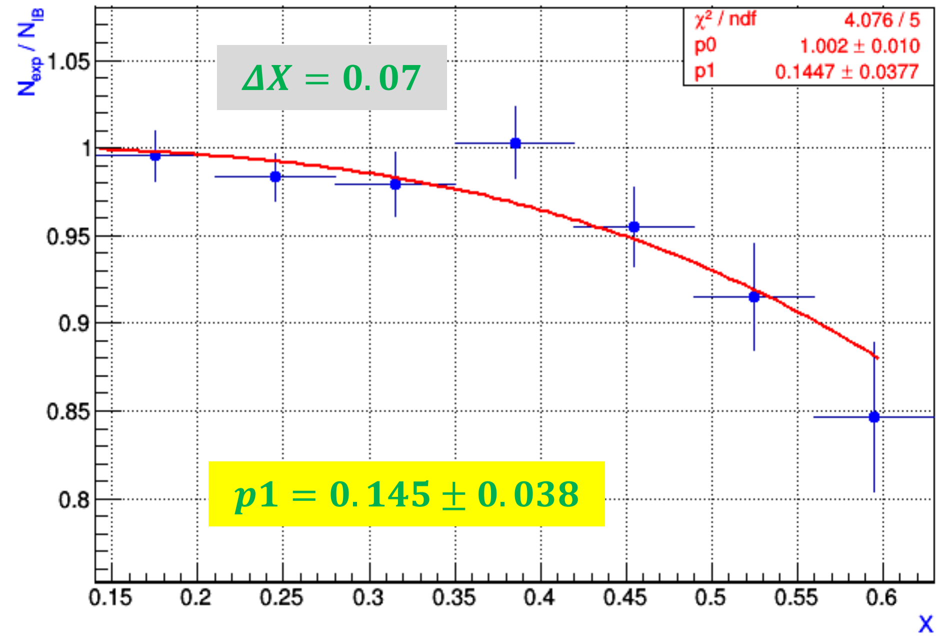


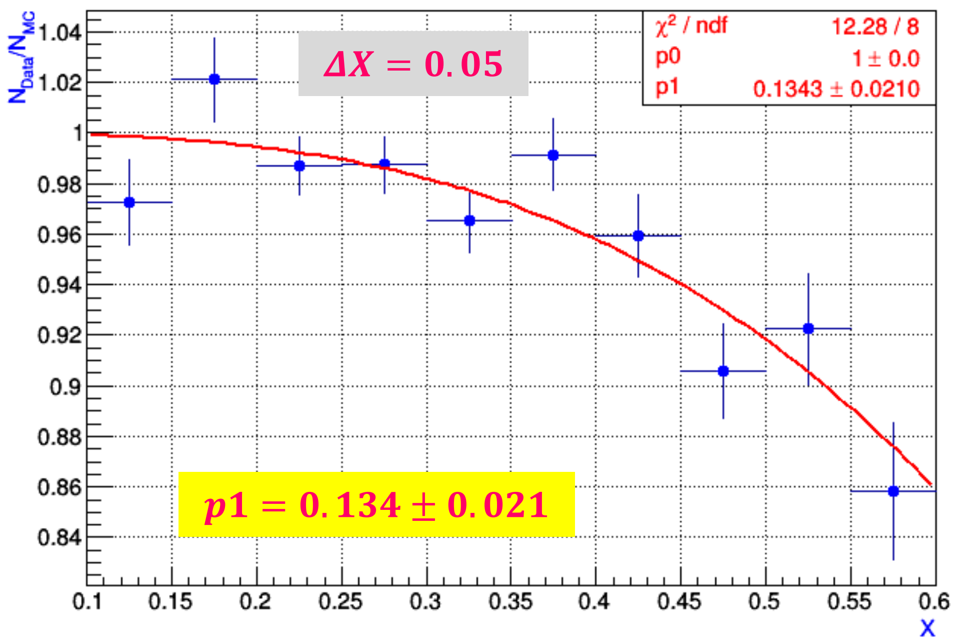


Both errors are negligible.

3) *Width of X-stripes*







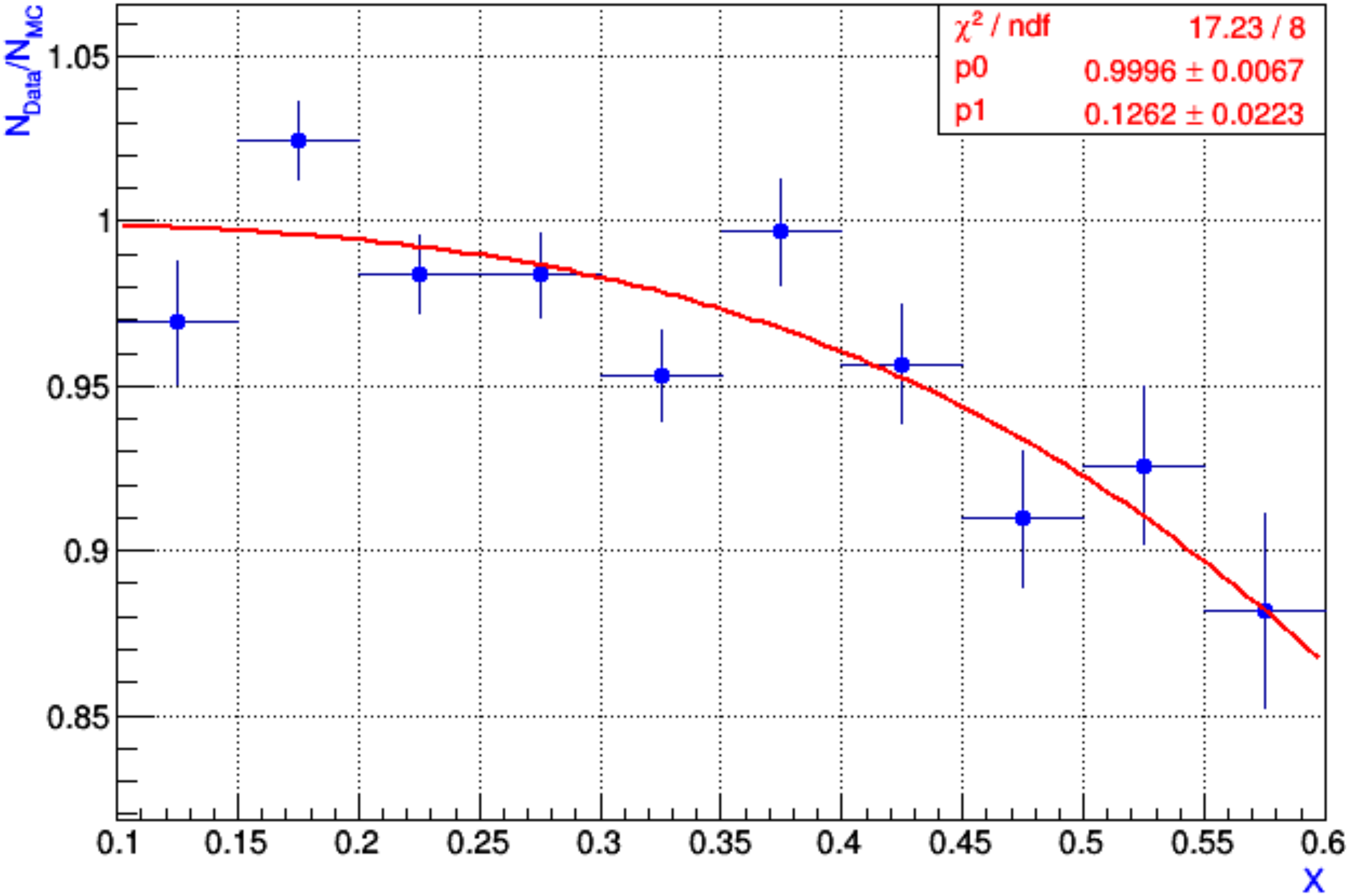
We repeated the data analysis procedure for 2 other values of **X**-binning:

- $\boldsymbol{\Delta X = 0.035}$, that is the worst **X**-resolution at maximal value of **X = 0.6**;
- $\boldsymbol{\Delta X = 0.07}$ **2 times higher** value.

$\boldsymbol{\sigma_X = 0.01}$

4) Y limits in X-stripes

FWHM cuts for selection of events were applied in **Y**-dependency for signal **MC**. Such cuts on **Y** are stronger than those used for main data analysis.

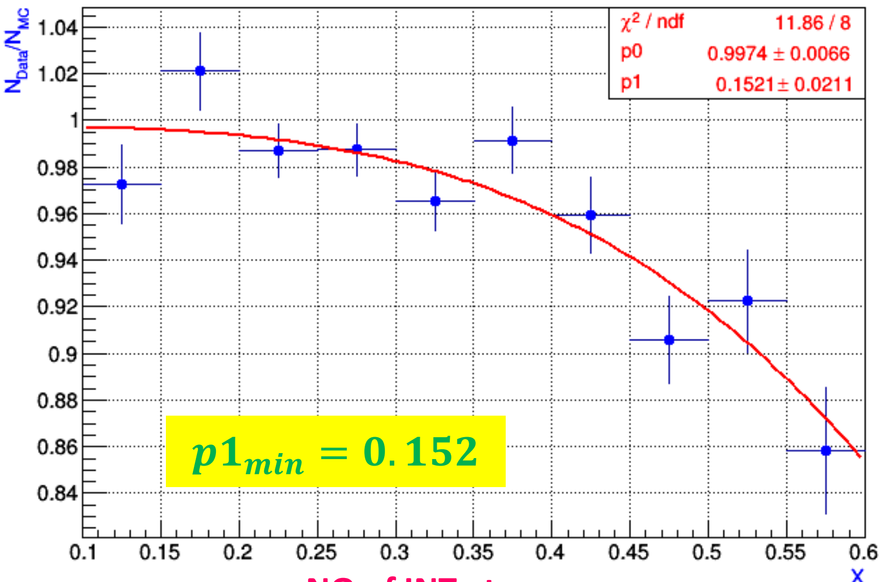


$$\sigma_X = 0.008$$

5) Possible contribution of *INT+* term

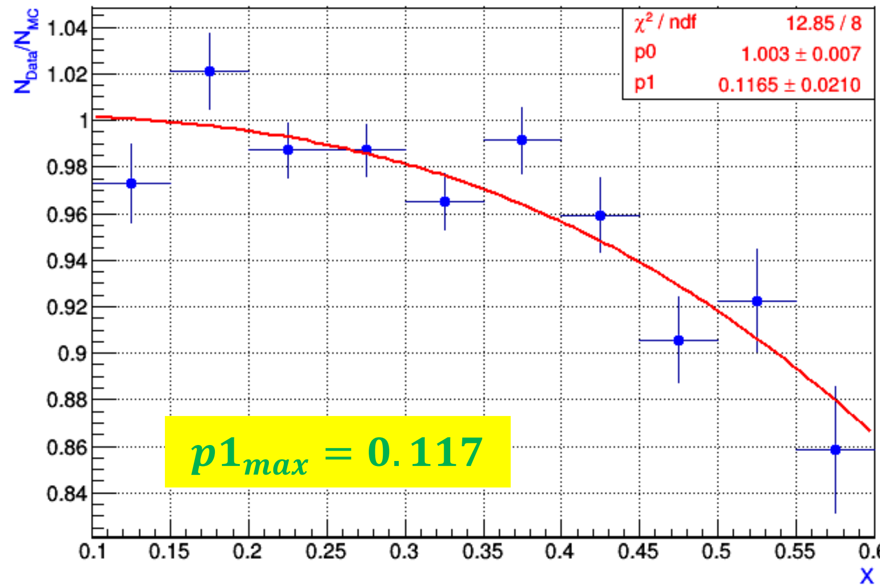
$$p_{signal} = p0(1 + (F_V + F_A)(f_{INT+}(x)/f_{IB}(x)) + (F_V - F_A)(f_{INT-}(x)/f_{IB}(x)))$$

Minimum of INT+ term



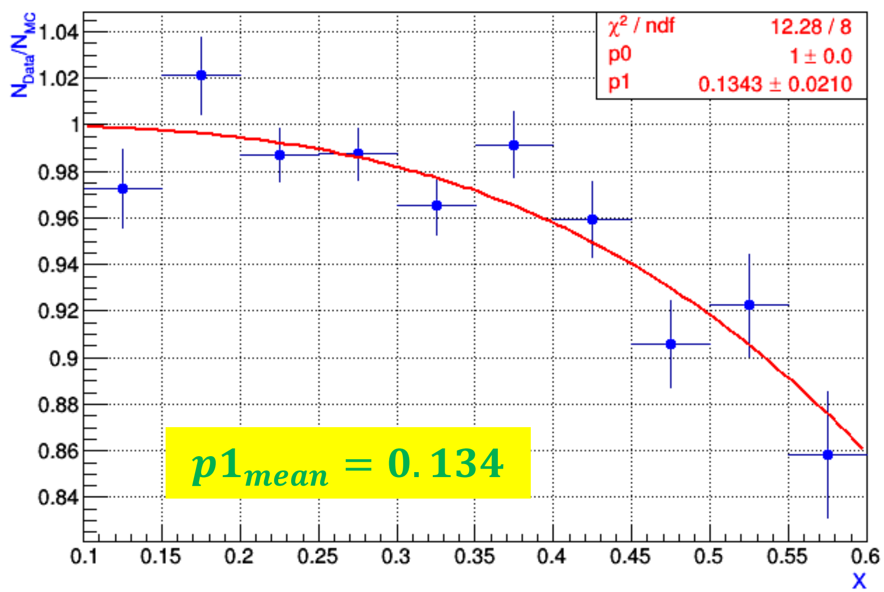
$p1_{min} = 0.152$

Maximum of INT+ term



$p1_{max} = 0.117$

NO of INT+ term



$p1_{mean} = 0.134$

$F_V + F_A$ value was measured by **E787** experiment **(Phys. Rev. Lett. 85 (2000) 2256)**.

$$|F_V + F_A| = 0.165 \pm 0.013$$

2 fits were repeated with minimal and maximal value of this measured sum.

$$\sigma_{INT+} = 0.018$$

Systematics

1) Non ideal description of signal and background in **MC** – $\mathbf{0.015}$
2) Left and right **X limits** (number of points in fit) – $\mathbf{0.0005}$
3) Width of X-stripes ($\Delta \boldsymbol{x}$ = 0.035 and 0.07 instead 0.05) – $\mathbf{0.01}$
4) Y limits in X-stripes (**FWHM** instead full signal reg.) – $\mathbf{0.008}$
5) Possible contribution of **INT+** term (**E787**) – $\mathbf{0.018}$

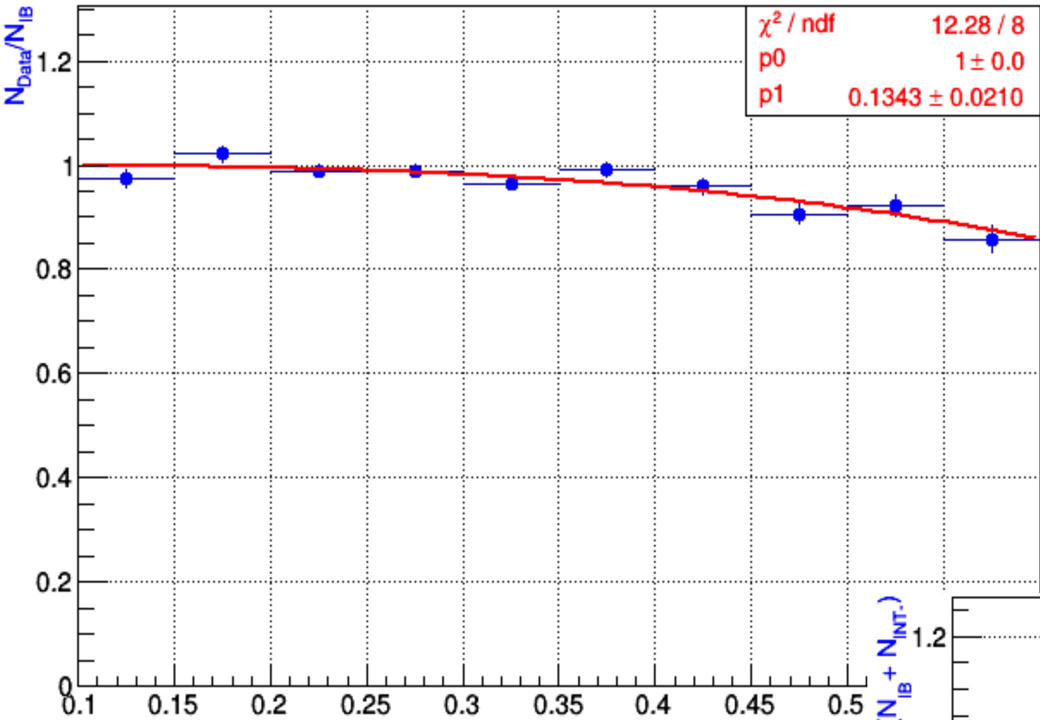
The total systematics from 5 possible sources is $\mathbf{0.027}$

Conclusion

1) Largest statistics about **95K** events of $K \rightarrow \mu\nu_{\mu}\gamma$ has been collected.
2) The negative **INT−** term has been extracted and $F_V - F_A$ has been measured: $\boldsymbol{F_V - F_A = 0.134 \pm 0.021(stat.) \pm 0.027(syst.)}$
3) The result is **2.4σ above** χPT O(p4) prediction.
   Resent calculation in framework of the gauged nonlocal effective chiral action (EχA) gives $\boldsymbol{F_V - F_A = 0.081}$ (**arXiv:1810.06815 [hep-ph], Oct 16 2018**).
   Our result is 1.6σ above EχA prediction.
4) The result is comparable within the errors with similar analysis of **ISTRA+** experiment : $\boldsymbol{F_V - F_A = 0.21 \pm 0.04(stat.) \pm 0.04(syst.)}$
5) **Measured stat. and syst. errors are ~2 times less than result of ISTRA+**

Backup slides

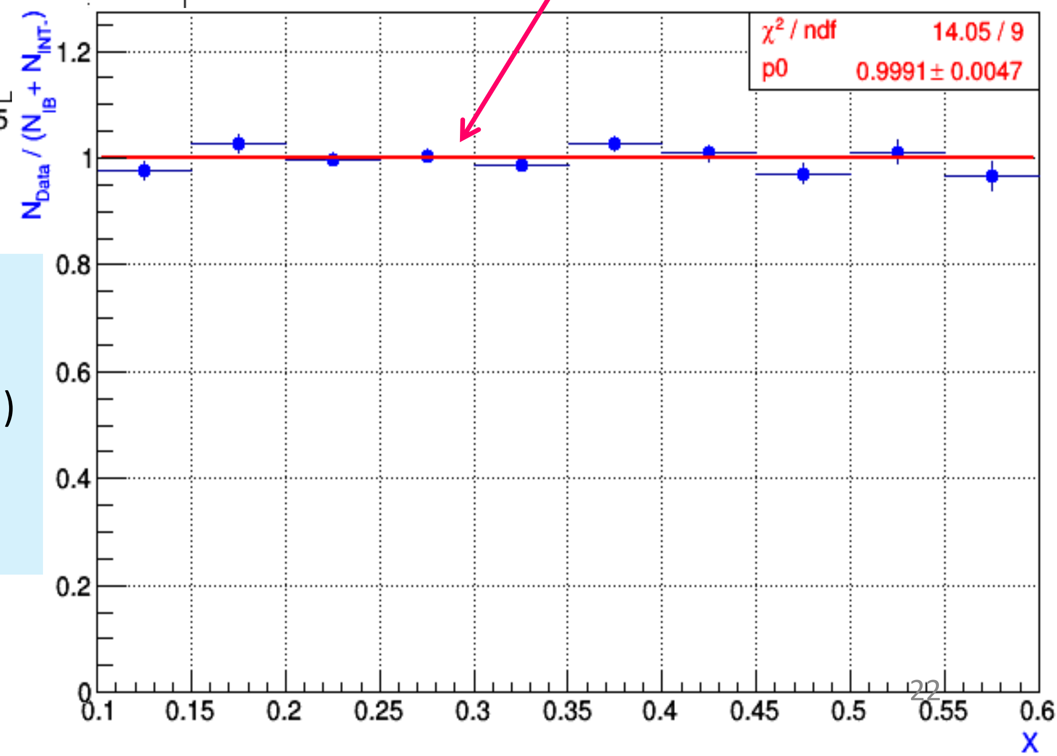
Verification of the fit method



Measured $p1 = F_V - F_A = 0.134$ value and normalization factor $p0 = 1$ were used to build new $\boldsymbol{p_{signal}}$ function

$$\boldsymbol{p_{signal}} = p0 \times \left(1 + p1 \times f\left(\frac{N_{INT-}}{N_{IB}}\right)\right)$$

Since the $\boldsymbol{N_{Data}/(N_{IB} + N_{INT-})}$ ratio does not depend on **X** the fit procedure is correct.



For additional checking the method we fit the original plot by

$$\boldsymbol{p_{signal}} = p0 \times \left(1 + (\mathbf{0.134} + p') \times f\left(\frac{N_{INT-}}{N_{IB}}\right)\right)$$

As we could suppose possible additional term $\boldsymbol{p' = 3 \times 10^{-5}}$ is about zero.