ПОЛЯРИЗУЕМОСТЬ ПИОНОВ:

ТЕОРИЯ и ЭКСПЕРИМЕНТ

М. А. Иванов (Дубна)

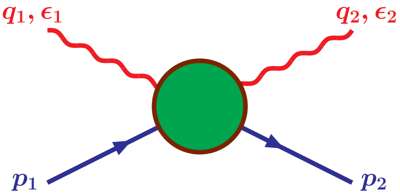
Марковские чтения

ИЯИ РАН, Москва, 15 мая 2015 года

Contents

Introduction

Pion polarizabilities: definition



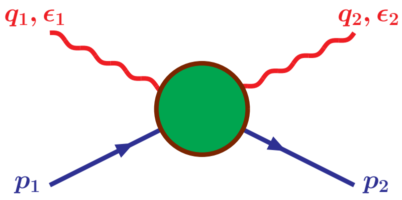
Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:

Introduction

Pion polarizabilities: definition



Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

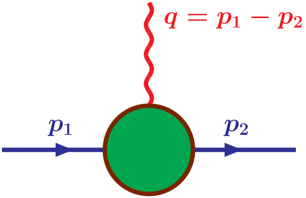
LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:

$$T_{\gamma\pi^+\to\gamma\pi^+} = \underbrace{-2\,e^2\,\vec{\epsilon}_1\cdot\vec{\epsilon}_2}_{\text{Born term}}$$

$$+ \quad 8\,\pi\,M_\pi \underbrace{\left\{\alpha_\pi\,\omega_1\omega_2\,\vec{\epsilon}_1\cdot\vec{\epsilon}_2 + \beta_\pi\,(\vec{\epsilon}_1\times\vec{q}_1)\cdot(\vec{\epsilon}_2\times\vec{q}_2)\right\}}_{\text{el-mag polarizabilities}} + \ldots$$

- The electric, α_π, and magnetic, β_π, polarizabilities characterize the response of hadrons to their two-photon interactions at low energies
- These quantities are analogous to electromagnetic radii and magnetic moments which characterize the response of hadrons to their single-photon interactions at low energies



$$< \pi(p_2)|j^\mu|\pi(p_1) >= e(p_1 + p_2)^\mu F_\pi(q^2)$$

$$F_\pi(q^2) = 1 + \frac{1}{6} r_\pi^2 q^2 + \dots$$

- The concept of the polarizability of molecules, atoms and nuclei was applied for the first time to hadrons in

  A. Klein, Phys. Rev. 99 (1955) 998,
  A.M. Baldin, Nucl. Phys. 18 (1960) 310,
  V.A. Petrun'kin, JETP 13 (1961) 804

- Many theoretical papers afterwards

- Only a few experiments

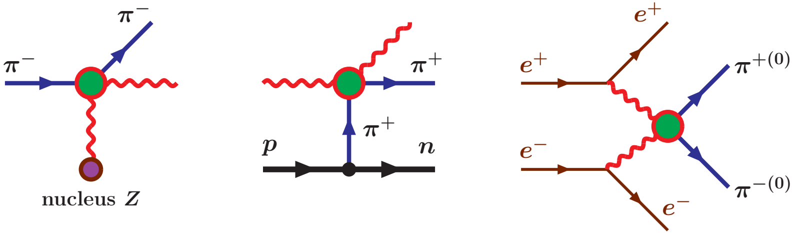
The units of measurement

As follows from the definition, the dipole pion polarizabilities are proportional to

$$\alpha_\pi(\beta_\pi) \sim \frac{\alpha}{M_\pi}\frac{1}{\Lambda^2} \approx 4 \times 10^{-4}\,\mathrm{fm}^3$$

where the hadronic scale $\Lambda \sim 4\pi F_\pi \sim 1\,\mathrm{GeV}$.
Then a natural choice of units for the dipole polarizabilities is $10^{-4}\,\mathrm{fm}^3$

Experiment



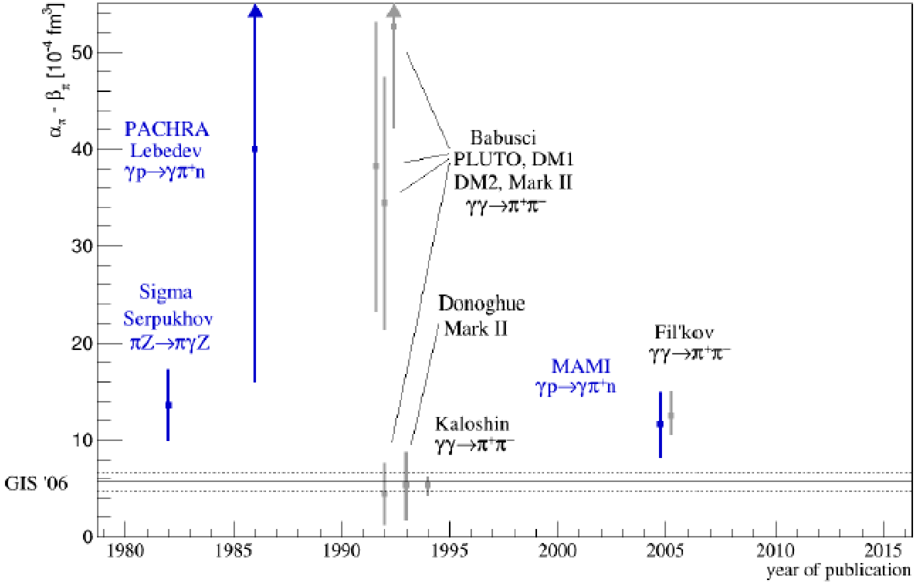
(a) (b) (c)

(a) **The scattering of high energy pions off the Coulomb field of heavy nucleus.**

(b) **The radiative pion photoproduction from the proton.**

(c) **The pion pair production in photon-photon collisions.**

Plot: T.Nagel, PhD TUM, 2012



GIS'06 = Gasser, Ivanov, Sainio, Nucl. Phys. B745 (2006) 84

General properties of pion polarizabilities

- **Classical sum rule** (Petrun'kin'64):

$$\alpha_\pi = \frac{\alpha}{3m} < r_\pi^2 > +2\alpha \sum_{n \neq 0} \frac{| < n|\mathcal{D}|0 > |^2}{E_n - E_0}$$

where $\mathcal{D}$ is the electric dipole operator.

$$\alpha_{\pi^\pm} \mapsto (3.5 - 6.8)$$

- The optical theorem relates the sum of polarizabilities to an unsubtracted forward dispersion relation

$$(\alpha + \beta)_\pi = \frac{M_\pi^2}{\pi^2} \int_{4M_\pi^2}^{\infty} \frac{ds'}{(s' - M_\pi^2)^2} \sigma_{tot}^{\gamma\pi}(s') > 0$$

Low Energy Theorem

- **Using current algebra/PCAC gives the relation of $\alpha_\pi(\beta_\pi)$ with the vector F_V and axial F_A structure constants for radiative pion decays $\pi^- \to e\nu\gamma$**

  (Terent'ev'73):

$$\alpha_{\pi^\pm} = -\beta_{\pi^\pm} = \frac{\alpha}{8\pi^2 F_\pi^2 M_\pi} \frac{F_A}{F_V} = 2.7 \pm 0.4$$

Update: PIBETA Coll.: PRL 103, 051802 (2009): = 2.78 (10)

Effective Lagrangians: $\mathcal{L}_{\rm QCD} \longrightarrow \mathcal{L}_{\rm eff}$ for $E << M_\rho$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$\mathcal{L}_{\rm eff}$ expressed in observed hadron fields,
has the same symmetry as QCD.

- The leading order in chiral SU(2) (pions and photons only):

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U\, D^\mu U^\dagger + M^2(U + U^\dagger)\rangle \,,$$

$$D_\mu U = \partial_\mu U - i(QU - UQ)A_\mu \,,$$

$$U \in SU(2)\,, \text{contains the pion fields}$$

- $M^2 = (m_u + m_d)B$
- F, B are low-energy constants (LECs) not fixed by chiral symmetry
- $\mathcal{L}_2$ - nonrenormalizable quantum field theory

- Higher order Lagrangians

$$\mathcal{L}_4 = \sum_{i=1}^{10} \ell_i K_i = \frac{\ell_1}{4} \langle D_\mu U D^\mu U^\dagger \rangle^2 + \cdots ,$$

$$\mathcal{L}_6 = \sum_{i=1}^{57} c_i P_i ,$$ (57→ 56) Haefeli, Ivanov, Schmid, Ecker 2007

- Local monomials K_i, P_i are known

Gasser, Leutwyler 1984,1985; Bijnens, Colangelo,Ecker 1999

- LECs ℓ_i, c_i absorb the divergences at order p^4 and p^6
- Notation later on: $\ell_i, c_i \to \ell_i^r, c_i^r$ UV finite parts of ℓ_i, c_i;
  $\ell_i^r, c_i^r \to \bar{\ell}_i, \bar{c}_i$ scale independent parts of ℓ_i, c_i.

- Higher order Lagrangians

$$\mathcal{L}_4 = \sum_{i=1}^{10} \ell_i K_i = \frac{\ell_1}{4} \langle D_\mu U \, D^\mu U^\dagger \rangle^2 + \cdots ,$$

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  $\ell_i^r, c_i^r \to \bar{\ell}_i, \bar{c}_i$ scale independent parts of ℓ_i, c_i.

- Calculations with $\mathcal{L}_{\mathrm{eff}}$ give an expansion in quark masses and external momenta.

Chiral perturbation theory (ChPT)

Gasser, Leutwyler 1984,1985

Pion polarizabilities in ChPT to one-loop

- Chiral expansion to one-loop

  Bijnens,Cornet 1988
  Donoghue,Holstein 1989

$$\alpha_\pi = -\beta_\pi = \frac{\alpha}{8\,\pi^2\,F_\pi^2\,M_\pi} \cdot \frac{1}{6}\,(\bar{\ell}_6 - \bar{\ell}_5)$$

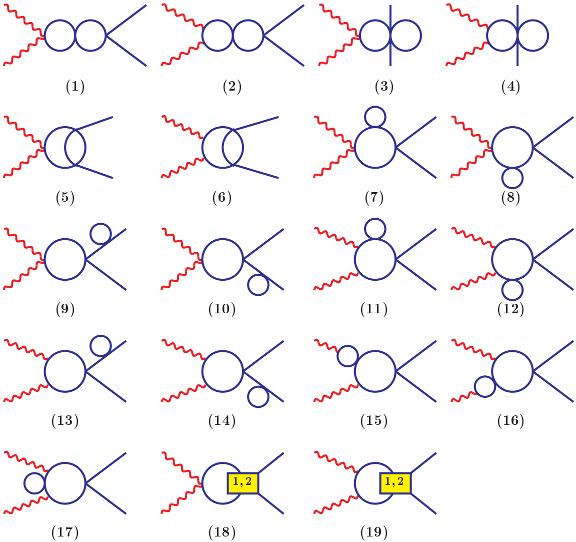
- The LECs $\bar{\ell}_{5,6}$ also arise in $\pi \to e\nu\gamma$-amplitude
- This gives a low energy theorem

  Terent'ev 1973

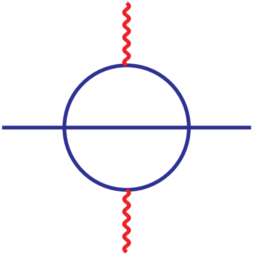
$$\alpha_\pi = -\beta_\pi = \frac{\alpha}{8\pi^2 F_\pi^2 M_\pi}\frac{F_A}{F_V}\left\{1 + O(M_\pi^2)\right\}$$

Pion polarizabilities in ChPT to two-loop

Gasser, Ivanov, Sainio 2005,2006



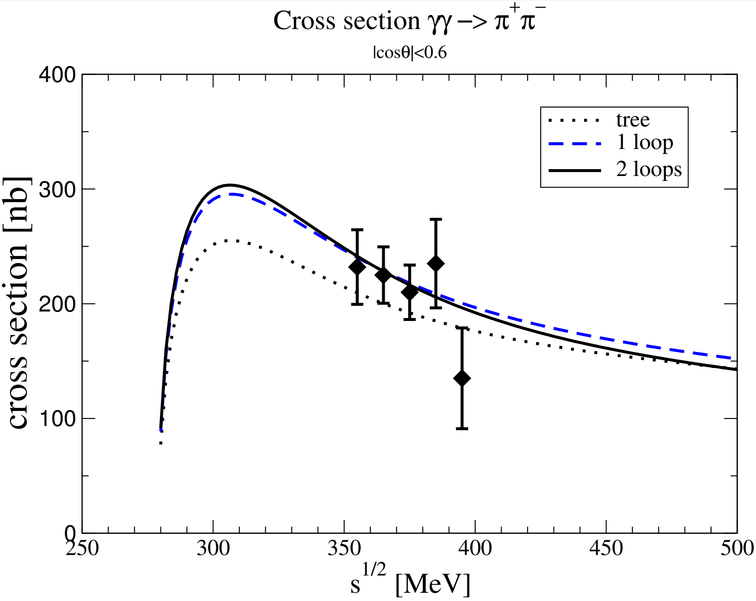
Acnode diagram



Invoke a dispersion relation for the function

$$\mathbf{I}(\mu, n; s) = \int_0^1 dx\,[x\,(1-x)]^n\;[1 - s\,x\,(1-x)]^\mu$$

$$= \int_4^\infty \frac{d\sigma\,\rho\,(\mu, n; \sigma)}{\sigma - s} \qquad (-1 < \mu < 0)$$



Experimental data from MARK II (SLAC) 1990

Charged pion polarizabilities: analytic results

$$(\alpha_1 \pm \beta_1)_{\pi^+} = \frac{\alpha}{16\,\pi^2\,F_\pi^2\,M_\pi}\left\{c_{1\pm} + \frac{M_\pi^2\,d_{1\pm}}{16\,\pi^2\,F_\pi^2} + O(M_\pi^4)\right\},$$

$$c_{1+} = 0\,, \qquad c_{1-} = \frac{2}{3}\,\bar\ell_\Delta\,,$$

$$d_{1+} = 8\,b^r - \frac{4}{9}\left\{\ell\left(\ell + \frac{1}{2}\,\bar\ell_1 + \frac{3}{2}\,\bar\ell_2\right) - \frac{53}{24}\,\ell + \frac{1}{2}\,\bar\ell_1 + \frac{3}{2}\,\bar\ell_2 + \frac{91}{72} + \Delta_+\right\}$$

$$d_{1-} = a_1^r + 8\,b^r - \frac{4}{3}\left\{\ell\left(\bar\ell_1 - \bar\ell_2 + \bar\ell_\Delta - \frac{65}{12}\right) - \frac{1}{3}\,\bar\ell_1 - \frac{1}{3}\,\bar\ell_2 + \frac{1}{4}\,\bar\ell_3 - \bar\ell_\Delta\bar\ell_4 + \frac{187}{108} + \Delta_-\right\}$$

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$$\Delta_+ = \frac{8105}{576} - \frac{135}{64}\,\pi^2 = \underbrace{-6.75,}_{\text{Burgi:}-8.69} \qquad \Delta_- = \frac{41}{432} - \frac{53}{64}\,\pi^2 = \underbrace{-8.08.}_{\text{Burgi:}-8.73}$$

Numerics

Numerical values of LECs

$$\bar{\ell}_1 = -0.4 \pm 0.6\,,\ \ \bar{\ell}_2 = 4.3 \pm 0.1\,,\ \ \bar{\ell}_3 = 2.9 \pm 2.4\,,\ \ \bar{\ell}_4 = 4.4 \pm 0.2$$

Colangelo, Gasser, Leutwyler 2001

$$\bar{\ell}_\Delta \doteq \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3\,.$$

Bijnens, Talavera 1997

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$$\bar{\ell}_\Delta \doteq \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3\,.$$

Bijnens, Talavera 1997

$$\begin{aligned} a_1^r &= -4096\pi^4\,(6\,c_6^r + c_{29}^r - c_{30}^r - 3\,c_{34}^r + c_{35}^r + 2\,c_{46}^r - 4\,c_{47}^r + c_{50}^r) \\ b^r &= -128\pi^4\,(c_{31}^r + c_{32}^r - 2\,c_{33}^r - 4\,c_{44}^r) \end{aligned}$$

Resonance ρ, a_1, b_1 exchange at $\mu = M_\rho$

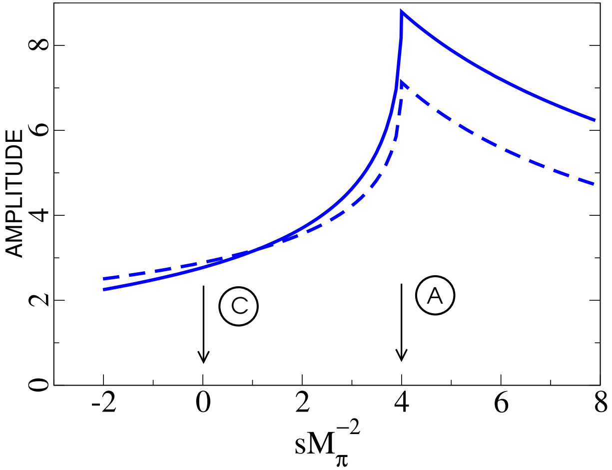
$$(a_1^r, a_2^r, b^r) = (-3.2, 0.7, 0.4)$$

ENJL model with large N_c (Bijnens & Prades)

$$(a_1^r, a_2^r, b^r) = (-8.7, 5.9, 0.38)$$

We use $b^r = 0.4 \pm 0.4$ and vary a_1^r from -10 to 0.

Chiral expansion at the Compton threshold



Example: spin non-flip amplitude. Solid line → two-loops, dashed line → one-loop

Charged pion polarizabilities

J. Gasser, M. A. Ivanov, M. E. Sainio, Nucl. Phys. B 745 (2006) 84-108

	ChPT to one loop	ChPT to two-loops
$(\alpha - \beta)_{\pi^+}$	6.0 ± 0.6	5.7 ± 1.0
$(\alpha + \beta)_{\pi^+}$	0	0.16 ± 0.14

Pion polarizability and JINR: retrospective review

Proposal to measure pion polarizability via Primakoff reaction

A.G. Galperin, G.V. Mitselmakher, A.G. Olshevski and V.N. Pervushin

Yad. Fiz. 32 (1980) 1053

- **The first observation of the Compton scattering off pion at SIGMA spectrometer.**
- **The first measurement of pion polarizabilities**
- **Dubna group brought their experience to the COMPASS experiment**

Main advantages of COMPASS

- **One can use pion and muon beams of the same momentum with the same setup configuration.**

$$\pi^-(\mathbf{A},\mathbf{Z}) \rightarrow \pi^-(\mathbf{A},\mathbf{Z})\gamma$$

$$\mu^-(\mathbf{A},\mathbf{Z}) \rightarrow \mu^-(\mathbf{A},\mathbf{Z})\gamma$$

- **Muon is the point-like particle and corresponding cross section for muon is known with high precision.**
- **So, muon data can be used as reference to control the systematics.**

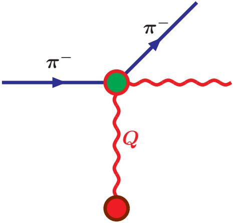
COMPASS

COmmon Muon Proton Apparatus for Structure and Spectroscopy

- COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at CERN in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams.
- Nearly 220 physicists from 13 countries and 24 institutions work in COMPASS.

COMPASS

C. Adolph *et al.* Phys. Rev. Lett. 114 (2015) 062002



Primakoff reaction:

$\pi^- + Z \to \pi^- + Z + \gamma$

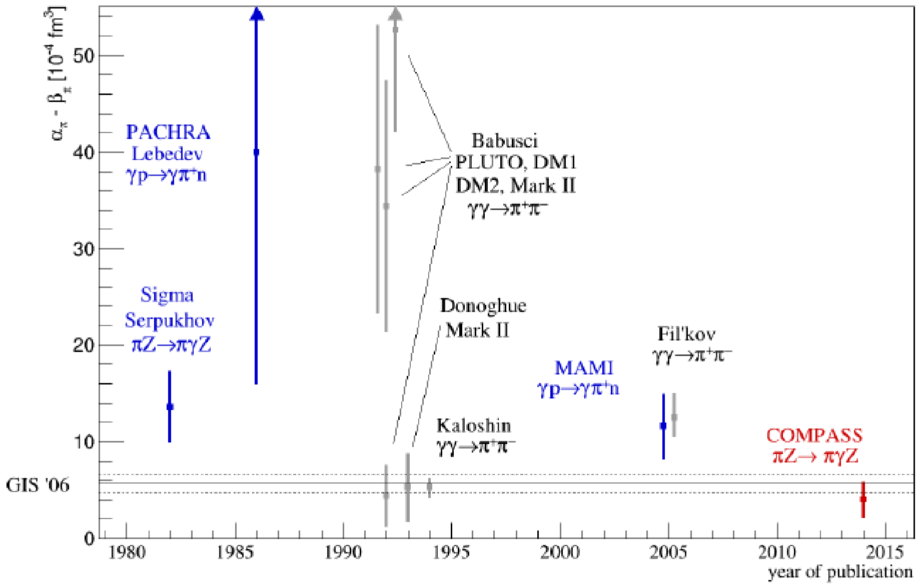
$E_\pi = 190$ GeV, $Q^2 < 0.0015$ GeV^2

$63 \cdot 10^3$ events

$\alpha_\pi = 2.0 \pm 0.6\,(\text{stat}) \pm 0.7\,(\text{syst})$

assumption: $\alpha_\pi + \beta_\pi = 0$

Plot: B. Badelek (COMPASS) 2015



COMPASS: C. Adolph et al. Phys. Rev. Lett. 114 (2015) 062002

Experimental information

Experiments		$(\alpha - \beta)_{\pi^\pm}$
$\gamma p \to \gamma \pi^+ n$	Mainz (2005)	$11.6 \pm 1.5_{\rm stat} \pm 3.0_{\rm syst} \pm 0.5_{\rm mod}$
L. Fil'kov, V. Kashevarov (2005)		$13.0^{+2.6}_{-1.9}$
$\gamma\gamma \to \pi^+\pi^-$	available data	
A. Kaloshin, V. Serebryakov (1994)		5.25 ± 0.95
$\gamma\gamma \to \pi^+\pi^-$	MARK II	
J.F. Donoghue, B. Holstein (1993)		5.4
$\gamma\gamma \to \pi^+\pi^-$	MARK II	
D. Babusci *et al.* (1992)		
$\gamma\gamma \to \pi^+\pi^-$	PLUTO	$38.2 \pm 9.6 \pm 11.4$
	DM 1	34.4 ± 9.2
	DM 2	52.6 ± 14.8
	MARK II	4.4 ± 3.2
$\gamma p \to \gamma \pi^+ n$	Lebedev Inst. (1986)	40 ± 24
$\pi^- Z \to \gamma \pi^- Z$	Serpukhov (1983)	$15.6 \pm 6.4_{\rm stat} \pm 4.4_{\rm syst}$
	COMPASS (2015)	$4.0 \pm 1.2_{\rm stat} \pm 1.4_{\rm syst}$

Summary

- ChPT is successful tool to analyse low-energy physics
- Chiral expansion for the $\gamma\gamma \to \pi\pi$ amplitude at the Compton threshold converges quite rapidly
- Two-loop result for the charged pion polarizability $(\alpha - \beta)_\pi$ is in agreement with very well known low-energy theorem
- However, there is a clash almost a factor of 2 (!) between this result and several experiments
- The last precise measurement of α_{π^+} performed by COMPASS is found in agreement with ChPT.
- Another experiments in CERN with JINR participation aiming to check the ChPT-predictions:

  DIRAC: $\pi\pi$-scattering lengths from $\pi^+\pi^-$– atom

  NA48: $\pi\pi$-scattering lengths from K $\to 3\pi$ – cusp

сообщения объединенного института ядерных исследований дубна

P2-83-83

В.А.Матвеев, А.В.Чижов

О МАГНИТОСТРИКЦИИ КВАРКОВОГО МЕШКА И ПОЛЯРИЗУЕМОСТИ АДРОНОВ

1983